

- (b) Solve the differential equation :

$$(x^2 - y^2) + 2xy \frac{dy}{dx} = 0.$$

- (c) Solve the differential equation :

$$x \frac{dy}{dx} + 2y = x^2 (x \neq 0).$$

7. (a) A family has two children. What is the probability that both the children are boys given that at least one of them is a boy?

- (b) Three cards are drawn successively, without replacement from a pack of 52 well shuffled cards. What is the probability that first two cards are kings and the third card drawn is an ace?

- (c) A man is known to speak truth 3 times out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9585

Roll No.

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B. Tech.

(Semester I) Theory Examination, 2011-12

ELEMENTARY MATHEMATICS-I

Time : 3 Hours

Total Marks : 100

Note : Attempt questions from each Section as per directions.

Section-A

Attempt *all* parts of this question. Each part carries 2 marks. 2×10=20

1. (i) What is the value of :

$$\lim_{x \rightarrow -1} [1 + x + x^2 + \dots + x^{10}]?$$

- (ii) What is the derivative of $\cos x$ at $x=0$?

- (iii) Is the function $f(x)=|x|$ continuous at $x=0$?

- (iv) Is Rolle's theorem applicable to the function $f(x)=(x-1)(x-4)e^{-x}$, $x \in [1, 4]$?

- (v) What is the value of :

$$\int \frac{1}{(x+a)^{1/2} + (x+b)^{1/2}} dx?$$

- (vi) What is the value of :

$$\int_0^{\pi/4} \sec^2 x dx?$$

- (vii) The differential equation :

$$xy \frac{d^2 y}{dx^2} + x \left(\frac{dy}{dx} \right)^2 - y \frac{dy}{dx} = 0$$

is of order _____ and degree _____.

- (viii) What is the form of the differential equation whose solution is $y = Ax + A^2$, A is a constant?

- (ix) In a throw of a die, the event of getting a prime number is given by _____.

- (x) Let E and F be the events such that $P(E) = 0.3$, $P(F) = 0.25$, $P(E \cap F) = 0.2$. What is the value of $P(E/F)$?

Section-B

Attempt any *three* parts of this question. $10 \times 3 = 30$

2. (a) Find the derivative of $f(x)$ from the first principle, where $f(x)$ is :

(i) x^2

(ii) $\sin x + \cos x$.

- (b) Prove that the curves $y^2 = x$ and $xy = c^2$ cut at right angles if $8c^4 = 1$.

- (c) Evaluate :

$$\int \frac{(3 \sin \theta - 2) \cos \theta}{5 - \cos^2 \theta - 4 \sin \theta} d\theta$$

- (d) Show that the differential equation :

$$(x - y) \frac{dy}{dx} = x + 2y$$

is homogeneous and solve it.

- (e) Three integers are chosen at random without replacement from the first 20 positive integers. Find the probability that their product is even.

Section-C

Attempt any *two* parts from each question. All questions are compulsory. $5 \times 2 \times 5 = 50$

3. (a) Suppose :

$$f(x) = \begin{cases} a+bx, & x < 1 \\ 4, & x = 1 \\ b-ax, & x > 1 \end{cases}$$

and if $\lim_{x \rightarrow 1} f(x) = f(1)$, what are possible values of a and b ?

- (b) Compute the derivative of $\tan x$ from the first principle.
(c) Find the derivative of :

$$f(x) = \frac{2x+3}{x-2}$$

from the first principle.

4. (a) Find $\frac{dy}{dx}$ when :

$$y = \tan^{-1} \left(\frac{3x-x^3}{1-3x^2} \right), \quad -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}.$$

- (b) Find the approximate value of $f(3.02)$, where $f(x) = 3x^2 + 5x + 3$.
(c) Show that semi-vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1} \left(\frac{1}{3} \right)$.

5. (a) Evaluate :

$$\int \frac{(x^2+1)e^x}{(x+1)^2} dx.$$

- (b) Evaluate :

$$\int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{\tan x}} dx.$$

- (c) Find the area of the region in the first quadrant enclosed by the x -axis, the line $y=x$ and the circle $x^2 + y^2 = 32$.

6. (a) Find the general solution of the differential equation :

$$\frac{dy}{dx} = \exp(x+y).$$